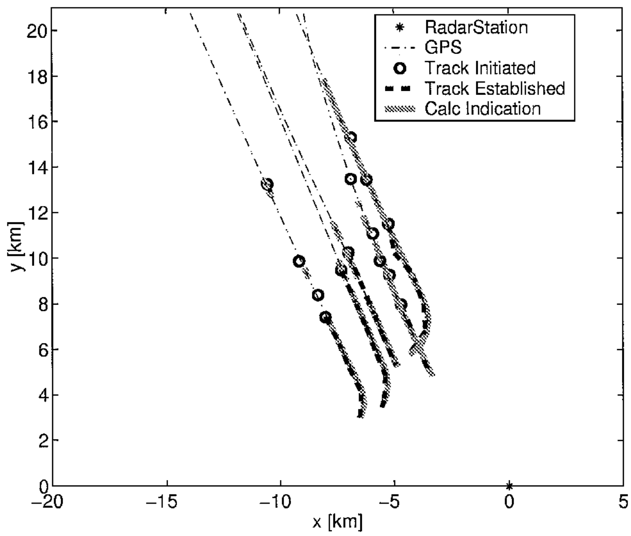

Fig. 3 Approaching the radar station at 850-m altitude.

Passing the radar station at 1000-m altitude results in flight times of 106 s and 60 s, respectively, which corresponds to a reduction of 43% in detection time. Hence, the reduction in the calculated detection time is possible to obtain in reality. This is achieved without any optimization methods applied.

Conclusions

The most important conclusion from the flight test is that substantial decrease in the time interval between the instant at which the aircraft is first detected by hostile radar and the instant at which the aircraft reaches a specified target is possible.

The current test suggests that the maximum RCS value over all radar frequencies should be used. To utilize successfully the computational models of RCS and radars, these models have to be updated and further verified, alternatively using measured data only. It may be possible to use a spline representation of the RCS for future use in optimization.

The example considering a Saab 105 in level flight approaching a radar station shows a reduction in detection time by almost 50%. The results in the flight tests show a difference in the detection time of 40%. This is obtained without optimization methods and shows the great potential in using flight-path optimization with radar range constraints. Future work will include a more general optimization formulation, where the radar range constraints are part of the trajectory optimization problem.

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Maximum Steady Roll Rate in Zero-Sideslip Roll Maneuvers of Aircraft

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I. Introduction

THE maximum steady roll rate achievable in response to an aileron input, in a maneuver where the sideslip is constrained to be zero, is a useful design parameter for combat aircraft. A larger value of the maximum roll rate, especially over the range of combat Mach numbers, is generally considered to be indicative of superior roll performance.[1] In practice, the zero sideslip constraint is implemented by using an aileron–rudder–interconnect law that suitably schedules the rudder as a function of the aileron deflection. It is commonly believed that the maximum achievable roll rate is limited by lack of aileron control power, or because the zero sideslip constraint cannot be enforced due to limits on the rudder deflection, or due to structural constraints.[2] However, for modern high-performance aircraft, nonlinear effects due to kinematic and inertial coupling (commonly called roll coupling) are dominant, and the maximum steady roll rate is usually decided by dynamic stability considerations. The purpose of this Note is to illustrate how the maximum steady roll rate in a zero-sideslip roll maneuver, in the presence of roll coupling nonlinearities, may be calculated using a continuation algorithm.

The problem of instabilities in rapid roll maneuvers of aircraft has been widely discussed in the literature ever since the phenomenon of roll coupling was first discovered by Phillips[3] in 1948. Hacker and Oprisiu[4] provided a review of the roll coupling problem, Schy and Hannah[5] interpreted the instability as a jump phenomenon, Carroll and Mehra[6] introduced the use of bifurcation analysis for prediction of roll-coupled instability, Jahnke and Culick[7] applied bifurcation theory to compute points of onset of instability for the F-14, and Ananthkrishnan and Sudhakar[8] suggested strategies to prevent jump in roll-coupled maneuvers. More recently, Goman et al.[9] have reviewed the use of bifurcation methods for nonlinear flight dynamics

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problems, including roll coupling, and Jahnke[10] has extensively discussed roll coupling instabilities of high-performance aircraft. However, none of these studies could accommodate maneuvers with constraints, such as level flight or zero sideslip. Roll maneuvers with zero sideslip and velocity vector roll constraints were studied by Ananthkrishnan and Sudhakar[11] and Modi and Ananthkrishnan,[12] respectively. Using a continuation algorithm, they could compute the achievable steady roll rate and the rudder deflection required to satisfy the constraint, for various values of the aileron deflection, but stability of the constrained states could not be determined using the algorithm. It is only very recently that an extended bifurcation analysis (EBA) procedure has been developed by the authors[13] that can simultaneously compute both the constrained states, as well as their stability. In the present Note, we implement the EBA procedure using the AUTO97 continuation algorithm[14] to solve the problem of determining the maximum steady roll rate in a zero-sideslip constrained roll maneuver. It is seen that the maximum steady roll rate corresponds to a transcritical bifurcation point beyond which the zero sideslip states are no longer stable.

II. Zero-Sideslip Roll Maneuver

Following standard practice, a set of seven equations (see Appendix) is used to describe the rate and attitude dynamics of an airplane in a rolling maneuver. These equations, along with the zero-sideslip constraint, are of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \delta a, \delta r, \delta e) \tag{1}$$

$$\beta = 0 \tag{2}$$

where $\boldsymbol{x}$ is the vector of states consisting of $(\alpha, \beta, p, q, r, \phi, \theta)$, and δa, δr, and δe are the aileron, rudder, and elevator deflections, respectively. The maneuver considered is a rolling pitch down initiated from a trim angle of attack corresponding to a 2-deg elevator deflection. The aircraft data used for this study are given in Table 1 and are the same as those considered by Schy and Hannah.[5] Pseudosteady-state (PSS) solutions are used to describe the steady-state response of the aircraft in the rolling maneuver. PSSs are steady-state solutions of the dynamic system in Eq. (1) with the effect of gravity neglected, that is, with (g/V) set to zero. This permits the equations for the roll angle ϕ and pitch angle θ to be treated separately, leaving behind a set of five equations in the variables α, β, p, q, and r, which represent the fast dynamic modes of the aircraft. Note that the zero-gravity assumption is to be used only to compute the PSS solutions; numerical simulations for the time responses should always use the complete set of seven equations. PSS solutions have been shown to represent the aircraft dynamics in steady rolling maneuvers adequately,[4,10] and have been widely used in previous studies of inertia-coupled roll maneuvers.[5,8]

Table 1 Aircraft data

Parameter	Value
m	2718 kg
c	1.829 m
ρ	1.2256 kg/m^3
I_y	16809 kg · m^2
C_{L_α}	4.35
C_{l_β}	−0.081
C_{l_r}	0.0309
$C_{l_{\delta r}}$	0.0
C_{m_q}	−9.73
$C_{m_{\dot\alpha}}$	−2.1
C_{n_p}	0.0
$C_{n_{\delta a}}$	0.0
b	11.0 m
S	20.07 m^2
I_x	2304.9 kg · m^2
I_z	18436 kg · m^2
C_{y_β}	−0.081
C_{l_p}	−0.442
$C_{l_{\delta a}}$	−0.24
C_{m_α}	−0.435
$C_{m_{\delta e}}$	−1.07
C_{n_β}	0.0218
C_{n_r}	−0.0424
$C_{n_{\delta r}}$	−0.01

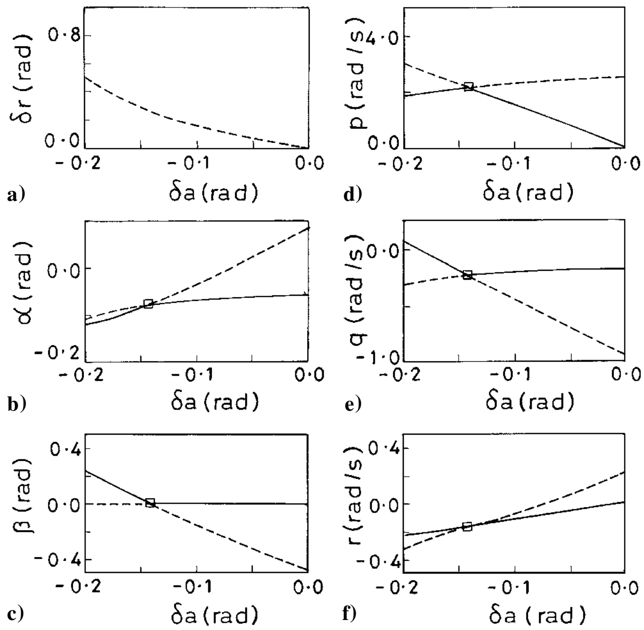

Fig. 1 Zero-sideslip roll maneuver: a) variation of rudder δr and b–f) PSS solutions for the state variables, as a function of aileron deflection δa.

The EBA procedure for the study of zero-sideslip roll maneuvers is implemented in the following two steps.

A. Computation of Rudder Schedule

In the first step, the rudder deflection required to enforce the zero-sideslip constraint is computed as a function of the aileron deflection. A limit of ±0.5 rad is assumed for both rudder and aileron deflections. PSS solutions of the aircraft dynamics in Eq. (1), with the zero-sideslip constraint in Eq. (2) satisfied, are computed using the AUTO97 algorithm. Computations are halted when either the rudder or aileron deflection limit is encountered. Elevator deflection is taken to be fixed at 2 deg. The rudder schedule thus computed is plotted in Fig. 1a. For convenience, only solutions for positive roll rates and negative aileron deflections are shown in Fig. 1. The required rudder deflection may be observed to vary nonlinearly with increasing values of aileron deflection, indicating that larger rudder deflections are necessary at higher roll rates to maintain zero sideslip in the presence of nonlinear roll coupling effects. It is seen that the rudder deflection limit of +0.5 rad is encountered for an aileron deflection of −0.2 rad, which is well short of the limiting aileron value of −0.5 rad. Thus, zero-sideslip roll maneuvers cannot be sustained for aileron deflections beyond −0.2 rad. However, the stability of the zero-sideslip PSS solutions remains to be established.

B. Determination of Stability and Bifurcations

In the second step, the stability of the constrained states, obtained with the rudder deflection schedule calculated in the first step, is determined. The rudder schedule is represented in terms of a function (typically, polynomial) of the aileron deflection as $\delta r = s(\delta a)$. The equations for the aircraft dynamics, with the preceding rudder schedule, now appear as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}[\boldsymbol{x}, \delta a, \delta r = s(\delta a), \delta e] \tag{3}$$

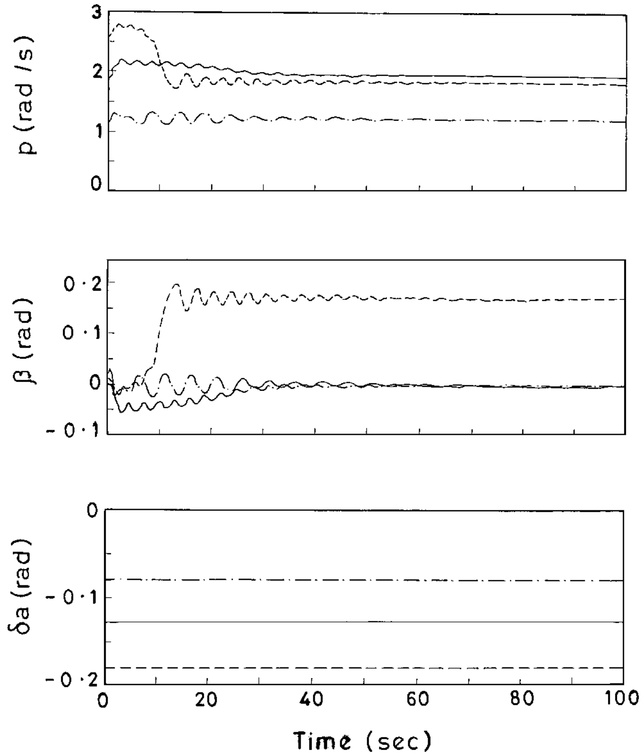

Fig. 2 Numerical simulations for different aileron deflections with the zero-sideslip rudder schedule in Fig. 1.

where, once again, the elevator deflection is maintained constant at 2 deg. PSS solutions of the dynamics in Eq. (3) are computed using the AUTO97 algorithm. Results of the computation for PSS roll, pitch, and yaw rates, and angles of attack and sideslip, are plotted Figs. 1b–1f, where full lines indicate stable PSS solutions and dashed lines indicate unstable PSS solutions. The PSS solutions with zero-sideslip constraint can be seen to lose stability at a transcritical bifurcation (indicated by an empty square) at an aileron deflection of approximately -0.14 rad. Beyond this value of aileron deflection, zero-sideslip roll maneuvers are dynamically unstable and, hence, cannot be sustained in practice. Referring to Fig. 1, the stable PSS solutions beyond the transcritical bifurcation point show positive sideslip, additional pitch up, and roll rates that are less than that at the transcritical point. The transcritical bifurcation point, thus, marks the maximum steady roll rate that can be achieved in a zero-sideslip roll maneuver. For the aircraft data in this study, this maximum roll rate can be read from Fig. 1 to be nearly 2.2 rad/s. This is to be contrasted with the PSS roll rate for the limiting value of rudder deflection, $\delta r = 0.5$ rad, which may be noticed from Fig. 1 to be about 3 rad/s. However, at this point, the PSS solutions are unstable, and the roll rate of 3 rad/s cannot be achieved in practice with zero sideslip. This is typical of most modern high-performance aircraft where the maximum steady roll rate is limited by dynamic stability considerations rather than control surface deflection limits.

Numerical simulations of the complete set of seven equations in Eq. (1) are carried out to confirm the predictions of the PSS bifurcation analysis in Fig. 1. Three different values of aileron deflection are used, as shown in Fig. 2, and the resulting roll rate and sideslip are also plotted in Fig. 2. Clearly, for the largest (negative) value of aileron deflection, the steady-state sidelip is nonzero, whereas for the two lower values of aileron deflection, zero steady-state sidelip is achieved. Also, for the largest (negative) value of aileron deflection, the steady-state roll rate is lower than that for the intermediate value of aileron deflection. The steady values of roll rate and sideslip in Fig. 2 are seen to match well with those predicted by the PSS bifurcation diagrams in Fig. 1 and attest to aileron deflections beyond the transcritical bifurcation point yielding roll maneuvers with nonzero sideslip and reduced steady roll rates.

III. Conclusions

The maximum steady roll rate in a zero-sideslip roll maneuver has been computed using the newly developed EBA procedure. It is shown that the maximum value of the roll rate is limited by roll coupling instabilities and is marked by a transcritical bifurcation point where the zero-sideslip states lose stability.

Appendix: Aircraft Dynamic Equations

$$\dot{\alpha} = q - p\beta + z_\alpha \alpha + (g/V)\cos\theta\cos\phi$$

$$\dot{\beta} = p\alpha - r + y_\beta \beta + (g/V)\cos\theta\sin\phi$$

$$\dot{p} = l_\beta \beta + l_p p + l_r r - qr(I_z - I_y)/I_x + l_{\delta a}\delta a + l_{\delta r}\delta r$$

$$\dot{q} = m_\alpha \alpha + m_{\dot{\alpha}}\dot{\alpha} + m_q q + pr(I_z - I_x)/I_y + m_{\delta e}\delta e$$

$$\dot{r} = n_\beta \beta + n_p p + n_r r - pq(I_y - I_x)/I_z + n_{\delta a}\delta a + n_{\delta r}\delta r$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

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